A Hybrid Fourth-Order FDTD Utilizing a Second-Order FDTD Subgrid

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***Abstract*—A hybrid method utilizing the second-order accurate in time and fourth-order accurate in space FDTD(2, 4) coupled with the standard second-order accurate both in time and space FDTD(2, 2) on a subgrid is presented. The accuracy of the method is tested by computing the S parameters of two monopoles mounted on a ground plane and it is found to be very satisfactory. Significant computational savings both in memory and time are accomplished by using this hybrid method.**

***Index Terms*—Finite-difference time-domain, higher-order schemes, subgrid modeling.**

I. Introduction

THE technological advancements of the last few decades have triggered new engineering problems and challenges. With the clock speed of all electronic equipment increasing, communication systems operate at higher frequencies. Therefore, the antenna elements become smaller, whereas the platforms they operate on, e.g., helicopter airframes, become electrically larger. These problems yield large computational domains and require significant computational resources, such as memory and execution time. Traditional finite methods (FDTD and FEM) are second-order accurate, thereby restricting the size of the domains that can be handled efficiently.

The standard FDTD method was introduced by Yee [1] in 1966. The classical FDTD method as proposed by Yee [1] is second-order accurate both in time and space [FDTD(2, 2)] thereby requiring many grid points per wavelength to accurately model the wave propagation. The FDTD method, as is typical for discrete methods, is dispersive; the phase velocity on the FDTD grid is not the same as the phase velocity of the physical continuous problem. In order to reduce dispersion errors, a finer discretization is required. On the other hand, finer discretizations demand larger memory and increased computational time, thereby restricting yet further the situations that can be tackled. Consequently, mesh refinement is not an efficient solution and sometimes is not even possible.

Numerous attempts have been made in the field of FDTD research to minimize phase errors [2]–[4]. One of the most promising approaches is based on higher-order accuracy schemes [5]–[9]. Such schemes theoretically exhibit lower dispersion errors and can utilize coarser grids as compared to those needed to achieve comparable levels of accuracy with a second-order scheme. Moreover, coarser meshes yield smaller computational spaces, reduced computational times and require less computational resources. Thus, ideally, the implementation of higher-order FDTD schemes will enable the efficient analysis of electrically larger problems.

The modeling of complex structures introduces additional challenges in high-order FDTD for the correct formulations of boundary conditions and discontinuities. The common method of dealing with these two issues is to implement one-sided higher-order finite differences. However, such one-sided stencils cause instabilities which are usually very difficult to resolve. Several papers have attempted to address these problems. In [10], time-stable boundary conditions for higher-order compact schemes were derived based on the summation-by-parts procedure, but only for one-dimensional hyperbolic problems. This approach was recently generalized to two-dimensional (2-D) problems in [11]. In [12], instabilities caused by one-sided high-order boundary conditions were resolved by using an artificial dissipation. In [13], a compact higher-order scheme was combined with boundary conditions implemented by one-sided differences, and a dissipative temporal integration method (Runge-Kutta fourth-order). In [14], the instabilities were eliminated through a filtering approach. A method of dealing with 2-D material discontinuities was presented in [15]. All of these methods, although promising, have not yet been verified for complex three-dimensional problems for which boundary conditions may be required, not only on the external boundary of the computational space, but also in the interior of the domain.

The proposed approach in this paper consists of combining a subgridding technique with a higher-order scheme. Subgridding techniques have been used in the past in the context of the standard FDTD [16], [17]. These methods divide the simulation space into two separate grids: a fine one and a coarse one. Here, the subgridding method of [16] is used in conjunction with the second-order accurate in time and fourth-order accurate in space FDTD(2, 4). On the fine grid, the standard FDTD(2, 2) is used to handle any of the fine features of the structure, whereas on the coarse grid, FDTD(2, 4) is used. Thus, existing successfully-applied techniques in FDTD(2, 2) for the incorporation of discontinuities, boundary conditions, and thin features are available for use on the fine grid. On the coarse mesh, away from phenomena associated with the complex structure, FDTD(2, 4) is

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used to mainly simulate the wave propagation in homogeneous media. Following this approach, high accuracy is obtained both around fine geometric features, such as thin wires, thin slots, etc., as well as in the wave propagation which is simulated by a higher-order scheme, i.e., FDTD(2, 4).

II. Method

The method developed here uses a modification of the subgridding technique of [16]. The boundary between the fine and the coarse grid is collocated with electric field components instead of magnetic field components. The ratio between the coarse and the fine grid cell sizes is chosen to be 1:3 as in [16]. This odd-ratio provides significant advantages, as discussed in [16]. A brief description of the procedure is given as follows (for details see [16]).

1) Apply FDTD(2, 4) on all the main grid points (including the ones inside the fine grid) and obtain $H^{n+1/2}$ and E^{n+1}.
2) Apply FDTD(2, 2) on the fine grid to obtain $h^{n+1/6}$.
3) Apply FDTD(2, 2) on the fine grid to obtain $e^{n+1/3}$. Update $e^{n+1/3}$ on the fine-coarse boundary using space and time interpolation. Apply (11) of [16] to weight $e^{n+1/3}$ one cell inside the fine grid.
4) Apply FDTD(2, 2) on the fine grid to obtain $h^{n+3/6}$. Use (12) and (13) of [16] to weight $h^{n+3/6}$ and $H^{n+1/2}$ collocated one coarse grid cell into the fine domain.
5) Update E^{n+1} at the fine-coarse boundary using the obtained values for $H^{n+1/2}$ from step 4.
6) Repeat step 3 to obtain $e^{n+2/3}$, step 2 to obtain $h^{n+5/6}$, and step 3 again to obtain e^{n+1}. Correct the fine-coarse grid boundary values of e^{n+1} using space interpolation of E^{n+1}.
7) Transfer all fine grid e field values to the corresponding collocated coarse grid E field values.

Capital letters E and H represent coarse grid field values, and small letters e and h represent fine grid field values.

III. Results

To illustrate the accuracy of the hybrid FDTD(2, 4)-Subgrid FDTD(2, 2), a geometry of two monopole antennas mounted on a finite ground plane is analyzed. The specifications of the geometry are shown in Fig. 1, and the radius of the two monopoles is 0.603 25 mm. The distance between the two monopoles was set to 32 cm so that it would be electrical large at the frequencies of interest [i.e., 32 cm is approximately 19 wavelengths (λ) at 18 GHz].

Three test simulations were performed by using 1) FDTD(2, 2) with a cell size of 4 mm (or $\lambda/4$ at 18 GHz), 2) FDTD(2, 2) with a cell size of 1.67 mm (or $\lambda/10$ at 18 GHz), and 3) the hybrid FDTD(2, 4)-Subgrid FDTD(2, 2) with a coarse grid cell size of 4 mm (or $\lambda/4$ at 18 GHz) and a fine grid cell size $4/3$ mm $\simeq 1.334$ mm (or $\lambda/12$ at 18 GHz). These three simulations are labeled 1, 2, and 3, respectively. Notice that in simulation 3, a fine grid was applied around each wire and the rest of the space used a coarse grid. The fine grid extended two coarse grid cells (or six fine grid cells) around each wire. The radius of the monopoles was taken into account in all three simulations both along the wire (using a thin wire model) and the excitation (using a source based on the radial electric fields). The S parameters were computed by using the procedure described in [18]. To speed the simulation times, all sources used an internal resistance of 50 ohms [18]. The results of the three simulations are illustrated in Figs. 2 and 3, where the S_{11} and S_{12} parameters of the two monopoles are compared against measurements

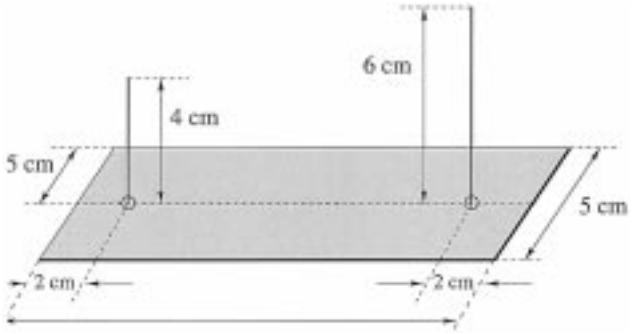

Fig. 1. Geometry of two monopoles on a ground plane.

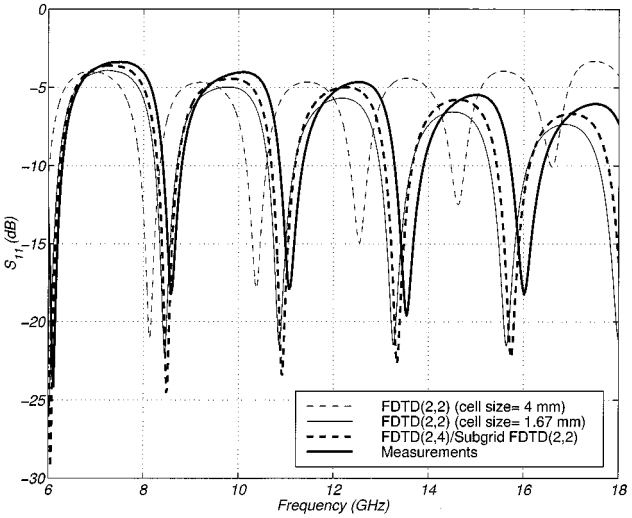

Fig. 2. S_{11} of the 6 cm long monopole shown in Fig. 1.

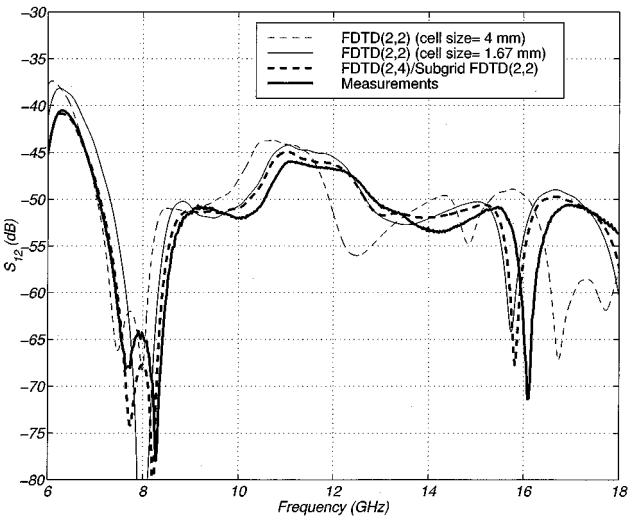

Fig. 3. S_{12} between the two monopoles shown in Fig. 1.

TABLE I
SIMULATION TIMES AND MEMORY REQUIREMENTS

Case	Simulation Time (min)	Memory (Mbytes)
1	15	3.5
2	118	14.0
3	48	3.9

performed in the Electromagnetic Anechoic Chamber facility (EMAC) at Arizona State University (ASU). S_{11} represents the reflection coefficient of the longest monopole (6 cm long) and S_{12} represents the coupling between the two monopoles. The accuracy of S_{11} is governed by the modeling of the wire and the discretization near the wire whereas the accuracy of S_{12} is governed by the accuracy of the FDTD stencil used to simulate the wave propagation from one element to the other. For both S_{11} and S_{12}, the computations of simulation 1 [FDTD(2, 2) with $\Delta x = 4$ mm] do not agree well with the measurements due to the poor discretization (only $\lambda/4$ at 18 GHz). However, simulations 2 and 3 provided predictions that are in very good agreement with measurements. Simulation 2 used FDTD(2, 2) and $\Delta x = 1.67$ mm (or $\lambda/10$ at 18 GHz), which is a good discretization of our structure. Simulation 3 used the hybrid FDTD(2, 4)-Subgrid FDTD(2, 2) and provided very good results for S_{11} since the discretization of the wires was done with a fine grid ($\Delta x = 4/3$ mm or $\lambda/12$ at 18 GHz). Furthermore, the hybrid FDTD(2, 4)-Subgrid FDTD(2, 2) computations for S_{12} were also accurate since the wave propagation from one element to the other was performed using the higher-order stencil FDTD(2, 4), with $\Delta x = 4$ mm (or $\lambda/4$ at 18 GHz). Therefore, it can be concluded that in the hybrid approach the field variations around thin geometric features, such as wires, are captured using the standard FDTD(2, 2) and the available sub-cell models on a fine grid. Moreover, the field propagation for large distances is accurately modeled by a higher-order FDTD stencil [FDTD(2, 4) in our case] on a coarse grid.

For the computation of S parameters, two simulations must be performed for each of our cases. The simulation times, as well as the memory requirements for each simulation, for cases 1, 2, and 3, are depicted and compared in Table I. To obtain accurate results with FDTD(2, 2), a cell size of 1.67 mm had to be used (case 2). This case required almost two and a half times more time and three and a half times more memory than the respective time and memory of case 3, which used the hybrid FDTD(2, 4)-Subgrid FDTD(2, 2). It should be noted that the computational savings will be significantly larger in cases where the computational domain is electrically large in all three directions (in our geometry, see Fig. 1, the domain is electrically large along one direction only).

IV. CONCLUSION

An accurate hybrid FDTD(2, 4) coupled with FDTD(2, 2) on a subgrid has been presented. The results indicate great computational savings both in memory and time. Moreover, this hybrid approach is very promising for other practical situations because of the flexibility for the inclusion of all existing thin and sub-cell models with FDTD(2, 2). Simultaneously, the method offers the high accuracy of FDTD(2, 4) for the propagation of waves over electrically large distances. Finally, this hybrid approach was found to be stable after several thousands of time-steps when the procedures of [16] were incorporated.

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